Abstract

Achieving the goal of limiting global warming to 1.5°C by 2050 requires worldwide net-zero CO2 emissions, and carbon pricing is a key tool in achieving this (OECD, 2021). Countries that have implemented carbon pricing policies have seen a slowdown in CO2 emissions (Khalifa et al. 2023, WEAI). To further this effort, we propose a theoretical and empirical application of the Vickrey Price Auction (VPA) algorithm for implementing efficient carbon pricing globally. The algorithm calculates the allocation of carbon permits by soliciting bids from each firm, resulting in a simple, transparent, and efficient process.

Introduction

Countries that have adopted carbon-pricing mechanisms have had a significant positive impact. Carbon pricing, which refers to a price put on CO2 and other greenhouse gas (GHG) emissions, maybe a policy tool for mitigating such emissions (Hepburn, 2007). By pricing GHG emissions, emitters have the option of paying for emissions or reducing them, which creates a window for them to minimize their compliance costs (PMR, P. 2021 and World Bank, 2022). Carbon pricing provides a mechanism for exchanging carbon permits between firms, enabling emission reductions to occur globally in the most cost-effective locations (OECD, 2021). Within this context, we provided a theoretical application of an algorithm for implementing efficient carbon pricing worldwide through the Vickrey Price Auction (VPA). The algorithm calculates the allocation of carbon permits after soliciting bids from each firm, making the process simple, transparent, and efficient.

Advantages and Disadvantages of Carbon Pricing tools

| Table 1 Summary of advantages and disadvantages of carbon pricing instruments |
|------------------|------------------|------------------|------------------|
| Carbon Tax     | Cap-and Trade    | Hybrid           |
| Certification  | Yes              | Yes              | Yes              |
| Certainty over the cost of emissions (i.e. price of carbon) | Yes | No | Some (possibly through price ceiling) |
| Certainty over emission reduction | No | Yes | Maybe |
| Incentives for cleaner technologies and innovation | High | High | High |

Revenue | High | High/Low depending on carbon Revenue allocation | Low |
Visibility | High | Moderate | Moderate |
Competitiveness impact/emissions leakage | It can be high depending on how carbon profile of the jurisdiction and the use of revenue | Can be mitigated depending on how permits are allocated and the use of resulting revenue | Can be mitigated depending on how permits are allocated and the use of resulting revenue |
Administrative and compliance requirements | Low | Moderate | Moderate |
Impact on poor consumers | High | Moderate depending on how carbon revenue is allocated | Moderate |
Non-fossil fuel emissions target | Low | High as it can easily benchmark all producing assets | Moderate |


ETS

In this type of policy, the government regulator imposes a cap on the total emissions by allocating permits to emit but then allows the permits to be traded, and a market price for emissions permits develops within the secondary market (Stavins, 2003). The idea is that the carbon emission level is predetermined by the regulator.
and assigned to various sectors. For more details, see (PMR, P. 2021 and World Bank, 2022).

- **Carbon Tax**

The government is typically the recipient of the tax revenues; according to World Bank data on carbon pricing, in 2022, 36 jurisdictions have implemented carbon taxes in (28) countries and (8) subnational jurisdictions (PMR, P. 2021 and World Bank, 2022). These initiatives cover (3 GtC02e), representing 5% of global GHG emissions in 2022. Notable countries include Canada, Finland, Sweden, Japan, Indonesia, South Africa, Chile, China, Colombia, the European Union (27 countries), Argentina, and Denmark.

- **Crediting Mechanisms**

Carbon crediting is issuing tradable emission reduction units to actors implementing approved emission reductions or removal activities (ICAP, 2021). Carbon pricing instruments such as carbon tax or ETS may allow carbon credit to complement it. In crediting mechanisms, actors who achieve emission reductions relative to a baseline or target get credit, which can be for specific projects, sector performance, or the result of policies (ADB, 2016).

- **Carbon Permit Auction Design**

The current marketplace for carbon credits is decentralized, where firms contact each other by their own means to trade their unused carbon credits. However, a critical disadvantage is that firms cannot know each possible trading opportunity because of a lack of information about firms and their willingness to sell their unused credits. This undermines the trade volume by making the market thinner than it otherwise would be. In other words, an otherwise beneficial trade may not occur between firms because of its decentralized feature, inducing a lack of information on firms as to market opportunities. Its monetary cost could be huge. Whenever a firm cannot procure a carbon credit, it must cut its production below its intended level. This causes an economic loss which is at least as much as the marginal benefit of increasing production. This is a lower bound for the loss because of the multiplier effect of the additional production in the economy at large. The total loss is at least as much as the total marginal benefits of additional productions by all firms that could not take place because of not utilizing all trade possibilities in the market.

To avoid the above problems and achieve efficient trades, we introduce a novel Carbon Permit Auction, where firms procure carbon permits to obtain a certain carbon emission amount. The amount of each allowance is irrelevant to the auction. Here how our auction works.

Let us suppose that \( k \) many permits are to be allocated among \( n \) many firms. The winning bids are the top \( k \) bids, where \( k \) is the total carbon permit supply in the auction. Each firm procures as many permits (credits) as its number of winning bids. If a firm obtains \( t \) many credits, it will pay the sum of the highest \( t \) losing bids.

We mathematically show that this auction design admits desirable economic properties: It yields efficient credit allocations and efficient prices so that the economy enjoys each beneficial trade. Moreover, we show that the firms’ best interest is to bid their true valuations, which makes the auction simple for firms (that is, firms do not need to strategize in the bidding)\(^3\). Below we formally introduce the auction design.

1. Each firm submits a bid for each additional permit it demands. For instance, if firm \( i \) only demands three permits, then it will submit three bids, say 10, 8, 5. These bids mean that firm \( i \) is ready to pay 10 for the first permit it obtains. It is ready to pay 8 for the second permit it gets. Likewise, it is ready to pay 5 for the third permit it obtains. These numbers mean that the firm is willing to pay 18 (10 + 8) for having two permits and 23 (10 + 8 + 5) for having three permits. Note that we can assume that the firm is bidding 0 for the number of permits beyond three.

2. This is a sealed-bid auction in the sense that firms do not know others’ bids while they submit their own bids. That is, each firm individually submits its bids through an online portal.

3. Once each firm submits its bids, the auction will terminate. To find how many permits each firm obtains and for which price, the auctioneer does the following.

- Order all the bids in decreasing order, and the first \( k \) (meaning that the highest \( k \) bids) are the winner, and the rest are the losers.

\(^2\) What are carbon markets and why are they important? | Climate Promise (undp.org)

\(^3\) The mathematical theory, prof algorithm and the platform for trading are available upon request
winning and losing bids. Note that the bids in each set are written in decreasing order.

- If a firm does not have a bid in \( W \), then it does not obtain any permit. It does not pay anything.
- If a firm has \( m \) bids in \( W \), then it gets \( m \) permits. In return to it, the firm pays the total of the highest losing \( m \) bids of the other firms. That is, it will pay the sum of the highest \( m \) losing bids of the other firms.

In what follows, we run the auction on a simple example to enhance our understanding of its working.

**Example 1.** Suppose there are three are three firms \( i, j, k \) and 6 permits. Let the bids be as follows;

\[
b_i = (50, 47, 45, 42, 15, 5); \quad b_j = (42, 28, 20, 12, 7, 3); \quad b_k = (45, 35, 24, 14, 9, 6);
\]

If we order the top 6 bids in decreasing order, we get \((50, 47, 45, 42, 40, 35)\). That is, \( W = \{50, 47, 45, 42, 40, 35\} \). On the other hand, we also order the losing bids in decreasing order, constituting set \( L \), as defined above. Hence,

\[
L = \{32, 28, 24, 20, 15, 14, 12, 9, 7, 6, 5, 3\}
\]

As firm \( i \) has three bids in \( W \), it obtains three permits. The firm pays the sum of the highest three losing bids of the other firms. Consider set \( L \) and identify the highest three bids of the other firms \( j, k \). These bids are \((28, 24, 20)\). Hence, firm \( i \) will pay \( 72 \) \((28 + 24 + 20)\). Firm \( j \) has only one bid in \( W \). Hence, it obtains only one bid. In return, it will pay the highest losing bid of the other firms. Hence, it will pay \( 32 \). Firm \( k \) has two winning bids \( W \), hence obtaining two permits. In return, it will pay the sum of the two highest losing bids of the other firms, that is, \( 32 + 28 = 60 \).

**Remark.** What if we have a critical tie (that is, one of them is to lose, and the other one is to be the winner)? To avoid it, at the beginning of the auction, we give numbers to the firms (may come from their submission times, the earlier a firm submits its bids, the lower number it receives), and we can resolve the ties by favoring the firms with lower numbers. For instance, consider two firms \( i \) and \( j \), and only one permit to be assigned. Let each firm bid the same, say 40. Then, each has the same bid; the question is, which firm will get the permit? If firm \( i \) submits its bid earlier, then it can be given number 1, and it will get the permit in return for the price of 40.

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**Policy recommendations/conclusions**

The summary suggests that there is no one-size-fits-all solution for market pricing in different economic sectors, and a combination of tools may be necessary. For instance, a carbon tax may be suitable for pricing carbon emissions in the transportation sector, while a carbon permit mechanism may be more appropriate for other CO2 emitters. The proposed algorithm can help allocate carbon permits by soliciting bids from firms, making the process transparent, efficient, and straightforward. Overall, the idea is to use a flexible approach that considers the unique characteristics of different industries to achieve effective market pricing for carbon emissions.

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